

# Analytical Modeling

\*Develop model \*Derive predictive solutions \*Identify key parameters \*Provide guideline in material design

## Residual Stresses in Multilayers & Graded Layers

- Decompose total strain into uniform and bending components, continuity condition at interface is satisfied automatically, model contains only three unknowns regardless of the number of layers, and exact closed-form solution is obtained.

### Multilayer



C. H. Hsueh, "Deformation of Multilayers due to Residual Stresses and Bending," *J. Appl. Phys.*, **91** [12] 9652–6 (2002).

C. H. Hsueh, L. C. De Jonghe, and C. S. Lee, "Modelling of Thermal Stresses in Joining Two Layers with Multi- and Graded-Interlayers," *J. Am. Ceram. Soc.*, **89** [1] 251–257 (2006).

### Graded Layer



C. H. Hsueh, "Stress Distribution and Curvature in Graded Semiconductor Layers," *J. Crystal Growth*, **258** [3–4] 302–309 (2003).

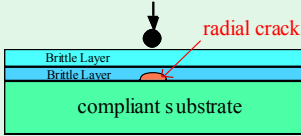
## Edge Delamination in Multilayers

- Derive exact closed-form solutions for interfacial peeling moment resulting from localized stresses normal to interface and interfacial shear force resulting from localized shear stresses in the edge region for each interface in multilayer.

C. H. Hsueh, *et al.*, "Interfacial Peeling Moments and Shear Forces at Free Edges of Multilayers," *J. Am. Ceram. Soc.*, **89** [5] 1632–38 (2006).

## Contact-Induced Radial Cracking

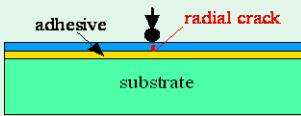
### Ceramic bilayer coating on compliant substrate



- Applications in microelectronic, structural, optical, and biological components.
- Solution predicts effects of layer thickness/properties on cracking.

C. H. Hsueh and P. Miranda, "Modeling of Contact-Induced Radial Cracking in Ceramic Bilayer Coatings on Compliant Substrates," *J. Mater. Res.*, **18** [5] 1275–1283 (2003).

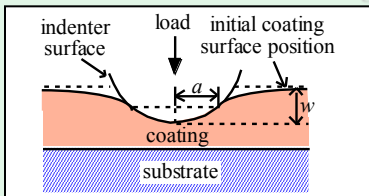
### Brittle layer on substrate with soft adhesive interlayer



- Applications in dental crowns, windshields, and other laminate structures.
- Solution predicts effects of adhesive interlayer on cracking.

C. H. Hsueh, J. H. Kim, and D. K. Kim, "Modeling of Effects of Adhesive Interlayers on Contact-Induced Radial Cracking in Brittle Coatings on Substrates," *J. Mater. Res.*, **18** [6] 1481–1486 (2003).

## Hertzian Indentation on Coating/Substrate Systems

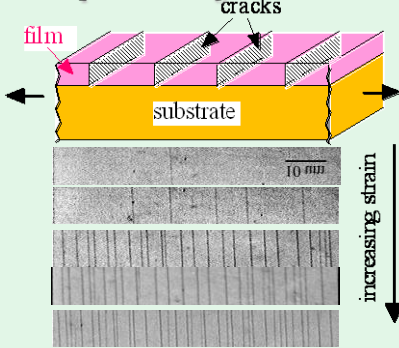


- Allow one to deconvolute elastic properties of coating from measured load-displacement relations.

C. H. Hsueh and P. Miranda, "Master Curves for Hertzian Indentation on Coating/Substrate Systems," *J. Mater. Res.*, **19** [1] 94–100 (2004).

C. H. Hsueh and P. Miranda, "Combined Empirical-Analytical Method for Determining Contact Radius and Indenter Displacement during Hertzian Indentation on Coating/Substrate Systems," *J. Mater. Res.*, **19** [9] 2774–2781 (2004).

## Multiple Cracking of Films on Substrates

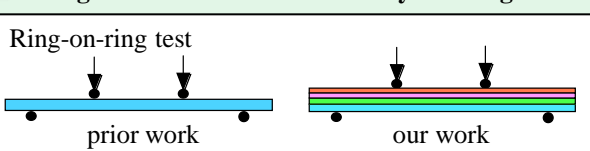


- As film/substrate is pulled unidirectionally, cracks transverse to the loading direction develop in the film.
- Solution predicts crack density as a function of applied strain, crack initiation strain (or film fracture energy), film thickness, and residual stress.
- If crack density–applied strain relation is measured at one film thickness, it can be used to predict those for other film thicknesses.
- If the residual stress in the film is unknown, the measured crack density–applied strain relation can be used to deduce this residual stress.

C. H. Hsueh and M. Yanaka, "Multiple Film Cracking in Film/Substrate Systems with Residual Stresses and Unidirectional Loading," *J. Mater. Sci.*, **38** [8] 1809–1817 (2003).

C. H. Hsueh and A. A. Wereszczak, "Multiple Cracking of Brittle Coatings on Strained Substrates," *J. Appl. Phys.*, **96** [6] 3501–3506 (2004).

## Strength Measurement of Multilayers using Biaxial Flexural Tests



C. H. Hsueh, M. J. Lance, and M. K. Ferber, "Stress Distributions in Thin Bilayer Discs Subjected to Ball-on-ring Tests," *J. Am. Ceram. Soc.*, **88** [6] 1687–1690 (2005).

C. H. Hsueh, C. R. Luttrell, and P. F. Becher, "Analyses of Multilayered Dental Ceramics Subjected to Biaxial Flexure Tests," *Dent. Mater.*, **22**, 460–469 (2006).

C. H. Hsueh, C. R. Luttrell, and P. F. Becher, "Modeling of Bonded Multilayered Disks Subjected to Biaxial Flexure Tests," *Int. J. Solids Struct.*, **43**, 6014–6025 (2006).

Existing analytical description of strength–fracture load relation is for monolithic materials only.